TRINITY COLLEGE

Year 12 Mathematics Specialist
Test 6 2019

Section 2 Calculator Assumed

Rectilinear Motion, Simple Harmonic Motion and Statistical Inference

STUDENT'S NAME Solutions

DATE: Monday 9 September **TIME:** 50 minutes **MARKS:** 50

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula page

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (3 marks)

Mr Presser samples 50 year 12 students. The mean height, μ of the sample is 172 cm and the standard deviation, $\cancel{\sigma}$ s is 11 cm.

Determine a 98% confidence interval for the sample mean.

0.98

$z = 2.3263$ ✓

$172 + 2.3263 \times \frac{11}{\sqrt{50}} \le \mu \le 172 + 2.3263 \times \frac{11}{\sqrt{50}}$ ✓

$168.38 \le \mu \le 175.62$ ✓

2. (9 marks)

Mr Jamieson wants to estimate the population mean number of minutes, μ , that male year 12 students read per week. He takes a random sample of 73 students and determines a 95% confidence interval for μ . The upper limit of this interval is 47.22 minutes and the lower limit is 42.78 minutes.

(a) Determine the sample mean for this sample of 73 students. [2]

μ is the middle of C.I.

$$\therefore \mu = \frac{47.22 + 42.78}{2} \checkmark$$

$$= 45 \checkmark$$

(b) Calculate, correct to 0.01 minutes, the sample standard deviation for the sample of 73 students. [3]

$$47.22 - 45 = k \frac{s}{\sqrt{73}} \checkmark \qquad k = 1.96 \checkmark \quad (95\% \text{ C.I.})$$

$$\Rightarrow \quad s = 9.68 \checkmark$$

(c) A student makes the following statements. Explain why each statement is either true or false.

(i) The probability that the sample mean lies in this confidence interval is 95% [2]

False. ✓

We are only 95% confident that interval contains the true mean. ✓

(ii) The amount of time that students spend reading is normally distributed because a distribution of sample means is normally distributed. [2]

False. ✓

We cannot make this assumption from the sample distribution. We do not know the distribution type of the population (C.L.T) ✓

3. (10 marks)

The mean wait time at a set of traffic signals has been observed to be normally distibuted with a mean $\mu = 60$ seconds and standard deviation $\sigma = 20$ seconds.

The wait times are recorded 100 times. Determine the probability that the:

(a) sample mean wait time will be between 57 seconds and 63 seconds. [3]

$$\bar{X} \sim N\left(60, \left(\frac{20}{\sqrt{100}}\right)^2\right)$$

$$P(57 \leq \bar{X} \leq 63) = 0.8664$$

Accuracy is required by the department managing the traffic signals. The mean wait time at the traffic signals is recorded a number of times.

(b) If the probablity for the mean wait time differs from μ by less than 5 seconds is 96%, determine n , the number of wait times that need to be measured. [3]

96% C.I $\Rightarrow$ $k = 2.0537$

$$\Rightarrow \quad 5 \leq 2.0537 \times \frac{20}{\sqrt{n}}$$

$$n \geq 67.49$$

$$n \geq 68$$

Daivik decides to test the validity of the department's claimed mean wait time. He records ten wait times on a stopwatch for a total time of 12 minutes. Daivik states that 'the departments figures for the average wait time are correct!'

(c) Perform the calculations necessary to comment on this claim. [4]

$$\bar{x} = \frac{12 \times 60}{10} = 72 \text{ sec}$$

95% C.I. $\quad 59.6 \leq \mu \leq 84.4$

99% C.I. $\quad 55.71 \leq \mu \leq 88.29$

Not confident with the claim. Even though both C.Is contain the mean, a sample of size 10 is not large enough and the C.Is are too wide.

4. (10 marks)

If, at any time t, a particle moves along a straight line with acceleration a cm/s^2, velocity v cm/s and displacement x cm from a fixed point O, the relationship between these variables is given by

$$a=\frac{dv}{dt}=\frac{d}{dt}\left(\frac{dx}{dt}\right)$$

(a) Use the chain rule to show that $\frac{d}{dx}(\frac{1}{2}v^2)=a$ [2]

$$LHS = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$= v.\frac{dv}{dx} \quad ✓$$

$$= v.\frac{dv}{dt}.\frac{dt}{dx} \quad ✓$$

$$= \cancel{v}.a.\frac{1}{\cancel{v}} \qquad = a$$

For the case when $a(t)=e^{2x}$ with initial conditions $x(0)=1$ cm and $v(0)=e$ cm/s, determine

(b) the velocity of the particle when $x(t)=5$ cm. [4]

$$a = v\frac{dv}{dx}$$

$$\Rightarrow \int e^{2x}\,dx = \int v\,dv \quad ✓$$

$$\Rightarrow \frac{1}{2}e^{2x} + c = \frac{v^2}{2}$$

$$\Rightarrow \qquad v^2 = e^{2x} + c_2 \quad ✓$$

at $t=0$ $x=1$ and $v=e$

$$\therefore \quad e^2 = e^{2(1)} + c_2 \quad \Rightarrow \quad c_2 = 0$$

$$\therefore \quad v^2 = e^{2x} \quad ✓$$

$$v = e^x$$

$$v\big|_{x=5} = e^5 \approx 148.41 \text{ cm/s} \quad ✓$$

(c) the time when the particle will have a displacement of 10 cm to the right of the origin. [4]

$$\frac{dx}{dt} = e^{x}$$

$$\Rightarrow \int e^{-x}\,dx = \int dt$$ ✓

$$\Rightarrow -e^{-x} = t + c$$ ✓

at $t = 0$ $x = 1$

$$\therefore -e^{-1} = c$$ ✓

$$\therefore t = e^{-1} - e^{-x}$$

$$t\Big|_{x=10} = e^{-1} - e^{-10}$$ ✓

$$\approx 0.3678 \text{ sec}$$

5. (11 marks)

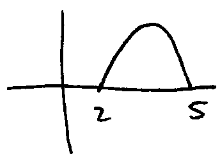
A particle moves along a straight line and its distance x metres from a fixed point O on the line after t seconds is given by

$$x = B\sin(kt - \theta)$$

where B , k and θ are positive constants and $\theta < 2\pi$.

The particle passes away through the point O for the first time after 2 seconds and away for the second time after 7 seconds. The maximum distance that the particle moves away from the point O is 10 metres.

(a) Determine the values of B , k and θ [5]



$T = 10\text{ sec} \Rightarrow 10 = \frac{2\pi}{k}$

$k = \frac{\pi}{5}$

$\text{Amp} = 10 \Rightarrow B = 10$

$\therefore\ x = 10\sin\left(\frac{\pi}{5}t - \theta\right)$

Sub in pt $x = 0$ when $t = 2$

$\Rightarrow 0 = \cancel{10}\sin\left(\frac{2\pi}{5} - \theta\right)$

$\Rightarrow 0 = \frac{2\pi}{5} - \theta$

$\therefore\ \theta = \frac{2\pi}{5}$

$\therefore\ x = 10\sin\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right)$

(b) When is the first time that the particle is furthest away from O ? [3]

$$\text{max dist} = 10 \quad \checkmark$$

$$\Rightarrow \quad \cancel{10} = \cancel{10}\sin\left(\frac{\pi}{5}t - \frac{2\pi}{5}\right)$$

$$\Rightarrow \quad \frac{\pi}{2} = \frac{\pi}{5}t - \frac{2\pi}{5} \quad \checkmark$$

$$\Rightarrow \quad \frac{9\pi}{10} = \frac{2\pi}{10}t$$

$$\Rightarrow \quad t = \frac{9}{2} \quad \text{sec} \quad \checkmark$$

(c) What is the maximum speed of the particle? [3]

$$v^2 = k^2(B^2 - x^2) \quad \checkmark$$

Max vel when particle passes through origin

$$\Rightarrow \quad v^2 = \left(\frac{\pi}{5}\right)^2(10^2 - 0^2) \quad \checkmark$$

$$= \frac{\pi^2}{25} \times 100$$

$$= 4\pi^2$$

$$\therefore \quad v_{max} = \pm 2\pi \quad m/s$$

$$\therefore \quad \text{Max speed} = 2\pi \quad m/s \quad \checkmark$$

6. (7 marks)

The displacement, $x(t)$ metres, of an object undergoing rectilinear motion is given by

$$x(t) = A\cos\omega t + B\sin\omega t$$

(a) Show that the object is undergoing simple harmonic motion. [2]

$$\dot{x}(t) = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$\ddot{x}(t) = -A\omega^2\cos\omega t - B\omega^2\cos\omega t$$

$$= -\omega^2(x(t))$$

$\therefore$ SHM

Initially, the object is located at $x = 1$ m and has velocity $v = 3$ m/s with period π metres.

$t = 0$

(b) Show that at time t, the position of the object is $x = \cos 2t + \frac{3}{2}\sin 2t$ [3]

$T = \pi$ m $\Rightarrow \pi = \frac{2\pi}{\omega}$

$\Rightarrow \omega = 2$

$x(0) = 1 \Rightarrow 1 = A\cos 0 + B\sin 0$

$A = 1$

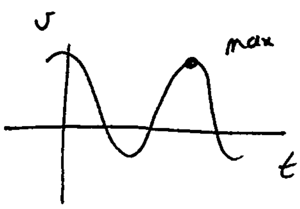
$v(0) = 3 \Rightarrow 3 = -A\omega\sin 0 + B\omega\cos 0$

$3 = B \times 2$

$B = \frac{3}{2}$

$\therefore x = \cos 2t + \frac{3}{2}\sin 2t$

(c) Determine when the object first has maximum velocity. [2]



Max $(2.8476, 3.606)$

$\therefore t = 2.85$ sec